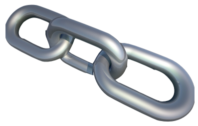
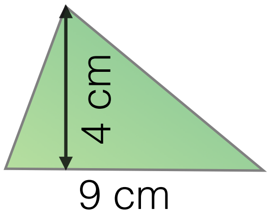
G16c Area of a triangle © BossMaths

Spot the links…



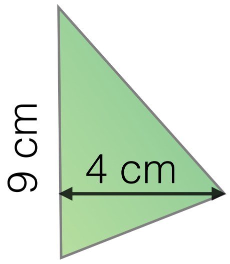
Find the area of each of the following triangles:

(1)



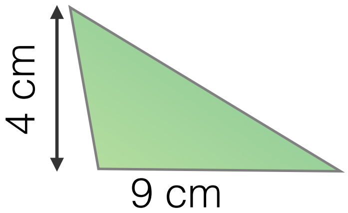
$\frac{9\times4}{2} = \underline{18\,cm^2}$

(2)



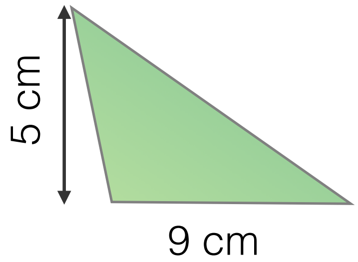
$\frac{9\times4}{2} = \underline{18\,cm^2}$

(3)



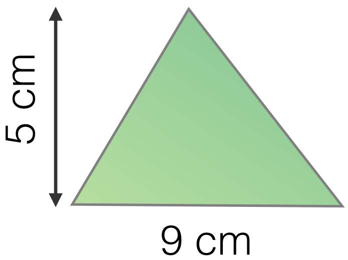
$\frac{9\times4}{2} = \underline{18\,cm^2}$

(4)



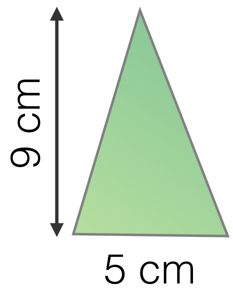
$\frac{9\times5}{2} = \underline{22.5\ cm^2}$

(5)



$\frac{9\times5}{2} = \underline{22.5\ cm^2}$

(6)



$\frac{5\times9}{2} = \underline{22.5\ cm^2}$

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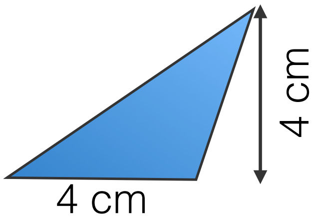
α

Alpha Exercise

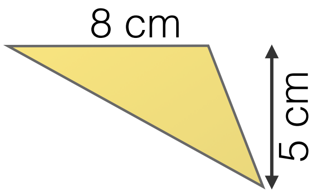
Find the area of each of the following triangles:

(1)



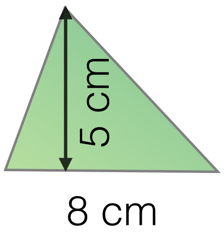
$$\frac{4 \times 4}{2} = \underline{8\text{ cm}^2}$$

(2)



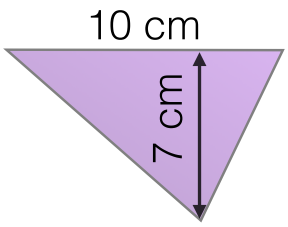
$$\frac{8 \times 5}{2} = \underline{20\text{cm}^2}$$

(3)



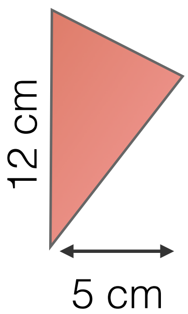
$$\frac{8 \times 5}{2} = \underline{20\text{cm}^2}$$

(4)



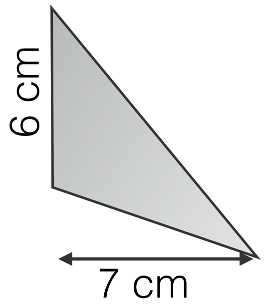
$$\frac{10 \times 7}{2} = \underline{35\text{cm}^2}$$

(5)



$$\frac{12 \times 5}{2} = \underline{30\text{cm}^2}$$

(6)



$$\frac{6 \times 7}{2} = \underline{21\text{cm}^2}$$

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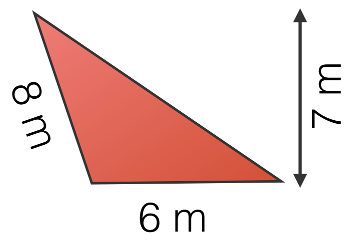
β

Beta Exercise

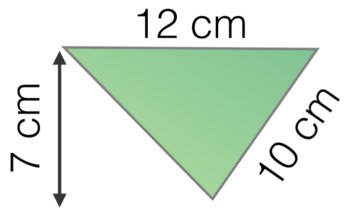
Find the area of each of the following triangles:

(1)



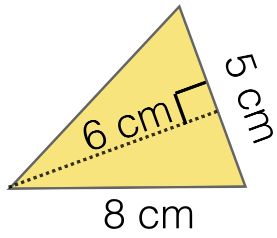
$$\frac{6 \times 7}{2} = \underline{21\ m^2}$$

(2)



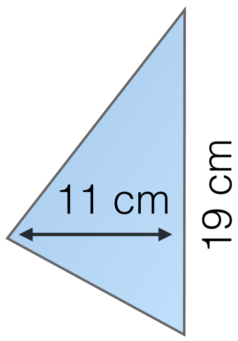
$$\frac{12 \times 7}{2} = \underline{42\ cm^2}$$

(3)



$$\frac{5 \times 6}{2} = \underline{15cm^2}$$

(4)



$$\frac{19 \times 11}{2} = \underline{104.5cm^2}$$

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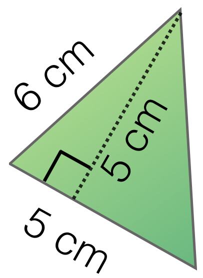
Gamma Exercise

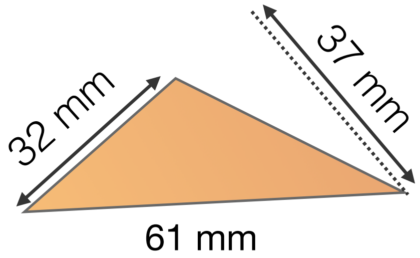
Find the area of each of the following triangles:

(1)



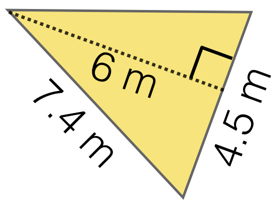
$$\frac{5 \times 5}{2} = \underline{12.5\,\text{cm}^2}$$

(2)



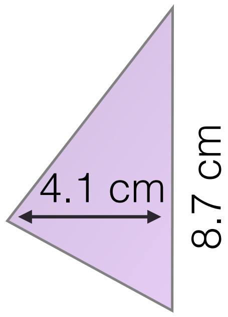
$$\frac{32 \times 37}{2} = \underline{592\,\text{mm}^2}$$

(3)



$$\frac{4.5 \times 6}{2} = \underline{13.5\,\text{m}^2}$$

(4)



$$\frac{4.1 \times 8.7}{2} = \underline{17.835\,\text{cm}^2}$$

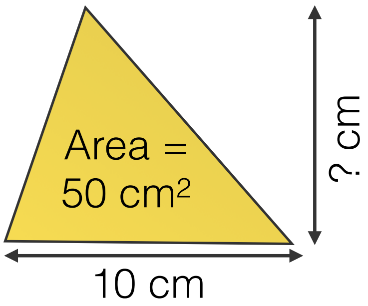
G16c Area of a triangle

Explain the mistake

Emma says that the missing length is 5 cm because 5 x 10 = 50 cm^2.

Emma is wrong.

Explain why.



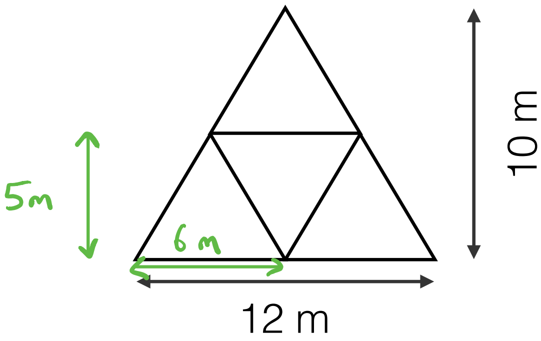
Emma seems to have forgotten that the area of a triangle is half the base multiplied by the height.

The correct height is 10 cm.

Exam-style question 1

Four identical triangles are tiled as shown to form one large triangle with a base of 12 metres, and a height of 10 metres, as shown in the diagram.

Work out the area of one tile.



Area of 4 tiles =

$$\frac{12 \times 10}{2} = 60 \, m^2$$

Area of 1 tile =

$$\frac{60}{4} = 15 \, m^2$$

Alternatively:

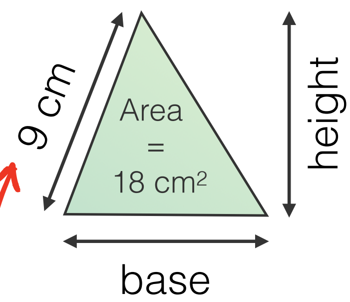
$$\frac{6 \times 5}{2} = 15 m^2$$

Exam-style question 2

Tyler draws a triangle whose base is equal to its perpendicular height.

The area of the triangle is 18 cm², and one of the sides is 9 cm long.

Find the base and height of the triangle.



Irrelevant

$$\text{Area} = 18\text{ cm}^2$$

$$\Rightarrow \frac{\text{base} \times \text{height}}{2} = 18$$

$$\Rightarrow \text{base} \times \text{height} = 36$$

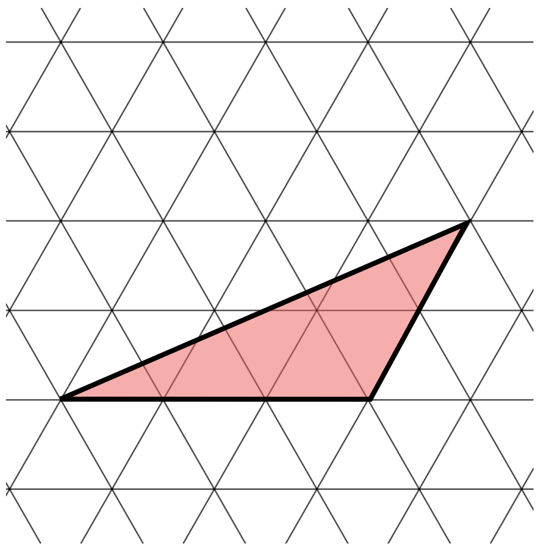
Since base = height,

$$\text{base} = \text{height} = \sqrt{36} = \underline{6\text{ cm}}.$$

Exam-style question 3

Here is a grid made up of equilateral triangles. Each small triangle has an area of 5 cm².

What is the area of the shaded triangle?



The triangle is half of a parallelogram made up of 12 small triangles, each with an area of 5cm².

$$\text{Area} = \frac{5 \times 12}{2} = \underline{30\text{ cm}^2}$$

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Challenge

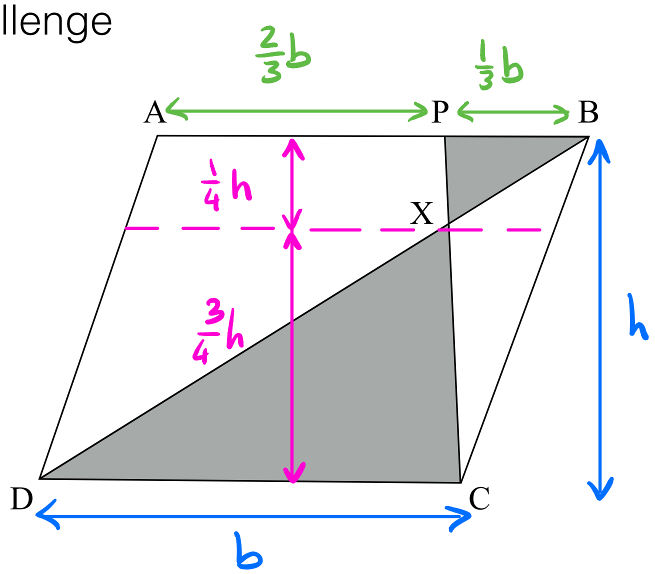
ABCD is a parallelogram.

The point P is the point on AB such that the ratio of AP : PB is 2 : 1.

PC and BD intersect at X.

Triangles XCD and PBX are shaded.

Show that the fraction of the parallelogram that is shaded is $\frac{5}{12}$.



Let ABCD have base b and height h as shown.
Then its area is bh.

Note ΔXCD and ΔXBP are similar, since

- $\angle PXB = \angle CXD$ (vertically opposite)
- $\angle PBX = \angle XDC$ (alternate)
- $\angle BPX = \angle XCD$ (alternate)

Now, $PB = \frac{1}{3} CD = \frac{1}{3} b$ i.e. ΔXCD is an enlargement of ΔXBP by scale factor 3.

Since the heights of ΔXCD and ΔXBP must sum to h, we must have heights of $\frac{3}{4}h$ and $\frac{1}{4}h$ respectively.

Hence shaded area = area of ΔXCD + area of ΔXBP

$$= \frac{1}{2} \times b \times \frac{3}{4}h + \frac{1}{2} \times \frac{1}{3}b \times \frac{1}{4}h$$

$$= \frac{3}{8}bh + \frac{1}{24}bh = \frac{10}{24}bh = \frac{5}{12}bh$$

i.e. $\frac{5}{12}$ of the area of ABCD.