A17b Part 2 Solving equations involving algebraic fractions where the unknown is on both sides of the equation

Warm-up activity

Solve the following equations:

1) $w+1=3w-17$

$1=2w-17$

$18=2w$

$9=w$

2) $11-2x=16+x$

$11=16+3x$

$-5=3x$

$\frac{-5}{3}=x$

3) $4y=7y-9$

$0=3y-9$

$9=3y$

$3=y$

4) $7k-2=13k+3$

$-5=6k$

$\frac{-5}{6}=k$

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Alpha Exercise

Solve the following equations:

1) $4a+3=\frac{2a}{7}$

$28a + 21 = 2a$

$26a = -21$

$a = \frac{-21}{26}$

2) $8n-13=\frac{n}{6}$

$48n - 78 = n$

$47n = 78$

$n = \frac{78}{47}$

3) $\frac{5x}{11}=-x+3$

$5x = -11x + 33$

$16x = 33$

$x = \frac{33}{16}$

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a

Alpha Exercise (contd.)

Solve the following equations:

4) $\frac{3}{2}u - 5 = -\frac{1}{2}u$

$3u - 10 = -u$

$4u = 10$

$u = \frac{10}{4}$

$u = \frac{5}{2}$

5) $\frac{8v}{5} = \frac{-4v}{5} + 6$

$8v = -4v + 30$

$12v = 30$

$v = \frac{30}{12}$

$v = \frac{5}{2}$

6) $\frac{-4(y+5)}{7} = \frac{12y}{7}$

$-4(y+5) = 12y$

$-4y - 20 = 12y$

$-20 = 16y$

$\frac{-20}{16} = y$

$y = \frac{-5}{4}$

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Alpha Exercise (contd.)

Solve the following equations:

7) $\frac{-2(p-5)}{4} = \frac{p}{4}$

$-2(p-5) = p$

$-2p + 10 = p$

$10 = 3p$

$\frac{10}{3} = p$

8) $4x - 11 = \frac{13x}{6}$

$24x - 66 = 13x$

$11x = 66$

$x = 6$

9) $\frac{4(t-2)+1}{12} = \frac{5}{12}t$

$4(t-2) + 1 = 5t$

$4t - 8 + 1 = 5t$

$4t - 7 = 5t$

$-7 = t$

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Beta Exercise

Solve the following equations:

1) $\frac{2}{3}x+4=\frac{1}{6}x$

$6\left(\frac{2}{3}x+4\right) = 6\left(\frac{1}{6}x\right)$

$4x + 24 = x$

$3x = -24$

$x = -8$

2) $\frac{2x+4}{3}=\frac{x}{6}$

$6\left(\frac{2x+4}{3}\right) = 6\left(\frac{x}{6}\right)$

$4x + 8 = x$

$3x = -8$

$x = \frac{-8}{3}$

3) $\frac{2x+4}{3}=\frac{x}{9}$

$9\left(\frac{2x+4}{3}\right) = 9\left(\frac{x}{9}\right)$

$6x + 12 = x$

$5x = -12$

$x = \frac{-12}{5}$

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Beta Exercise (contd.)

Solve the following equations:

4) $\frac{2x+4}{6}=\frac{x}{9}$

$18\left(\frac{2x+4}{6}\right) = 18\left(\frac{x}{9}\right)$

$6x+12 = 2x$

$4x = -12$

$x = -3$

5) $\frac{2(x+4)}{6}=\frac{x}{9}$

$18\left(\frac{2(x+4)}{6}\right) = 18\left(\frac{x}{9}\right)$

$6(x+4) = 2x$

$6x+24 = 2x$

$4x = -24$

$x = -6$

6) $\frac{2(x+4)}{6}=\frac{x+1}{9}$

$18\left(\frac{2(x+4)}{6}\right) = 18\left(\frac{x+1}{9}\right)$

$6(x+4) = 2(x+1)$

$6x+24 = 2x+2$

$4x = -22$

$x = \frac{-22}{4} = \frac{-11}{2}$

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Beta Exercise (contd.)

Solve the following equations:

7) $\frac{2(x+4)}{6}=\frac{x}{5}$

$$30\left(\frac{2(x+4)}{6}\right)=30\left(\frac{x}{5}\right)$$
$$10(x+4)=6x$$
$$10x+40=6x$$
$$4x=-40$$
$$x=-10$$

8) $\frac{2(x+4)+1}{6}=\frac{x}{5}-2$

$$30\left(\frac{2(x+4)+1}{6}\right)=30\left(\frac{x}{5}-2\right)$$
$$10(x+4)+5=6x-60$$
$$10x+45=6x-60$$
$$4x=-105$$
$$x=-\frac{105}{4}$$

9) $\frac{2(x+4)}{6}+1=\frac{x}{5}-2$

$$30\left(\frac{2(x+4)}{6}+1\right)=30\left(\frac{x}{5}-2\right)$$
$$10(x+4)+30=6x-60$$
$$10x+70=6x-60$$
$$4x=-130$$
$$x=-\frac{130}{4}=-\frac{65}{2}$$

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γ

Gamma Exercise

Solve the following equations:

1) $\frac{3}{p} = \frac{5}{2p+1}$

Multiply both sides by $p(2p+1)$

$3(2p+1) = 5p$

$6p + 3 = 5p$

$p = -3$

2) $\frac{8}{3t-1} = \frac{5}{t}$

Multiply both sides by $t(3t-1)$

$8t = 5(3t-1)$

$8t = 15t - 5$

$5 = 7t$

$\frac{5}{7} = t$

3) $\frac{2}{b} = \frac{-1}{b+7}$

Multiply both sides by $b(b+7)$

$2(b+7) = -1(b)$

$2b + 14 = -b$

$3b = -14$

$b = \frac{-14}{3}$

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Gamma Exercise (contd.)

Solve the following equations:

4) $\frac{5}{4-n}=\frac{7}{2n}$

Multiply both sides by $2n(4-n)$

$2n(5) = 7(4-n)$

$10n = 28-7n$

$17n = 28$

$n = \frac{28}{17}$

5) $\frac{3}{1-2d}=\frac{5}{2(d+6)}$

Multiply both sides by $2(d+6)(1-2d)$

$6(d+6) = 5(1-2d)$

$6d+36 = 5-10d$

$16d = -31$

$d = \frac{-31}{16}$

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Gamma Exercise (contd.)

Solve the following equations:

6) $\frac{2}{x}-\frac{5}{3x}=\frac{1}{x-1}$

Multiply both sides by $3x(x-1)$

$\Rightarrow \frac{6}{3x}-\frac{5}{3x}=\frac{1}{x-1}$

$x-1 = 3x$

$-1 = 2x$

$\Rightarrow \frac{1}{3x}=\frac{1}{x-1}$

$-\frac{1}{2} = x$

7) $\frac{2}{3(y+1)}=\frac{5}{7(5-y)}$

Multiply both sides by $21(y+1)(5-y)$

$14(5-y) = 15(y+1)$

$70-14y = 15y+15$

$55 = 29y$

$\frac{55}{29} = y$

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Gamma Exercise (contd.)

Solve the following equations:

8) $\frac{3}{1-5q} = \frac{8}{4(q+2)}$

Multiply both sides by $4(q+2)(1-5q)$

$12(q+2) = 8(1-5q)$

$12q + 24 = 8 - 40q$

$52q = -16$

$q = \frac{-16}{52} = \frac{-4}{13}$

9) $\frac{7}{15m} - \frac{3}{20m} = \frac{1}{5+m}$

$\Rightarrow \frac{28}{60m} - \frac{9}{60m} = \frac{1}{5+m}$

$\Rightarrow \frac{19}{60m} = \frac{1}{5+m}$

Multiply both sides by $60m(5+m)$

$19(5+m) = 60m$

$95 + 19m = 60m$

$95 = 41m$

$\frac{95}{41} = m$

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Delta Exercise

Given that each of the three equations has a solution of $\frac{7}{2}$, find whole numbers to fill in the blanks.

1) $\frac{7x}{5} + \boxed{3} = \frac{22+5x}{5}$

Substituting $x = \frac{7}{2}$, we get

$\frac{49}{10} + \square = \frac{79}{10}$

Therefore, the blank is $\frac{30}{10} = 3$

2) $-3x = \frac{\boxed{21}}{2} - 6x$

Multiplying both sides by 2, we get $-6x = \square - 12x$

Adding $12x$ to both sides, we see $6x = \square$

Substituting $x = \frac{7}{2}$, we see the blank is 21.

3) $\frac{2}{7} = \frac{\boxed{3}}{3x}$

Note that $\frac{2}{7}$ is the reciprocal of $\frac{7}{2}$.

Since $x = \frac{7}{2}$, $\frac{2}{7} = \frac{1}{x}$.

Hence $\frac{\square}{3x} = \frac{1}{x}$. Therefore, the blank is 3.

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Explain the mistake

Alice is trying to solve the following equation:

$$\frac{p}{7}+9=\frac{2p}{7}$$

Alice decides to multiply both sides of the equation by 7. She writes:

$$p+9 = 2p$$
$$9 = p$$

What mistake has Alice made?

$$7\left(\frac{p}{7}+9\right) = 7\left(\frac{2p}{7}\right)$$

$$\Rightarrow \quad p+63 = 2p$$

Alice wrote $p+9=2p$. It appears that she did not multiply the 9 by 7.

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Exam-style question

Solve the following equations:

a) $\frac{7+4x}{5} = \frac{2x+9}{3}$

$3(7+4x) = 5(2x+9)$

$21 + 12x = 10x + 45$

$2x = 24$

$x = 12$

b) $\frac{5}{7+4x} = \frac{3}{2x+9}$

$5(2x+9) = 3(7+4x)$ Same as above

$x = 12$

c) What do you notice?

The equations have the same solution.

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Challenge

The following equations have no solutions:

$$\frac{6}{7+3x}=\frac{4}{2x-5} \qquad \frac{9}{10+6x}=\frac{3}{2x+7} \qquad \frac{-4}{5x-1}=\frac{8}{3-10x}$$

a) Can you explain why they have no solutions?

$6(2x-5) = 4(7+3x) \Rightarrow 12x-30 = 28+12x$

$9(2x+7) = 3(10+6x) \Rightarrow 18x+63 = 30+18x$

$-4(3-10x) = 8(5x-1) \Rightarrow -12+40x = 40x-8$

All are of the form $ax+b = ax+c$, with $b \neq c$.
These have no solutions.

b) Come up with some equations involving algebraic fractions that also have no solutions.

e.g. $\frac{3}{2x+5} = \frac{12}{8x+7}$