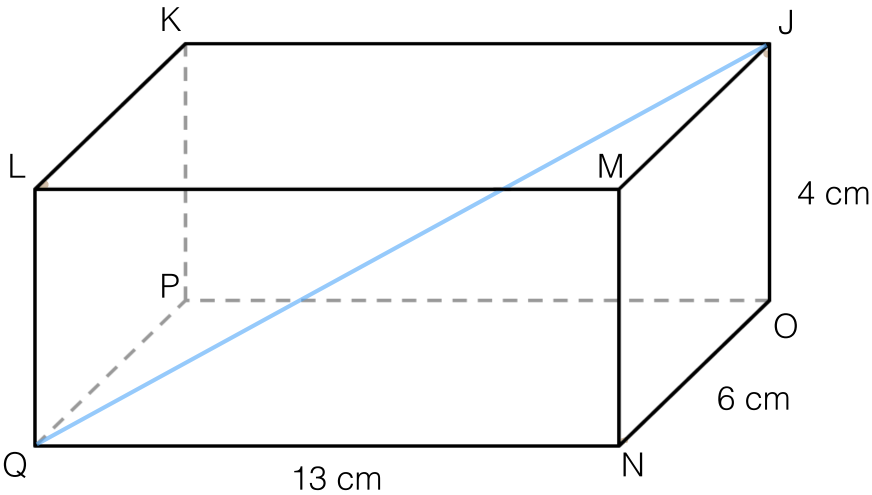
G20d Trigonometry with right-angled triangles in 3D shapes

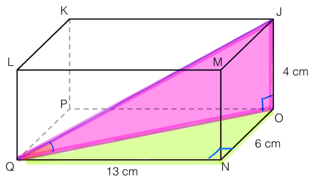
α

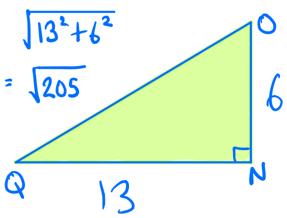
Alpha Exercise 1

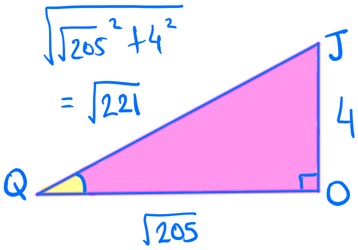
Answer the following correct to the nearest 0.1°



(a) Find the angle between JQ and the plane OPQR.



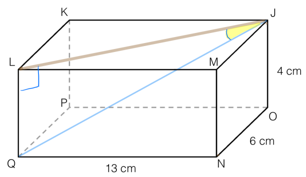




$$\angle JQO = \tan^{-1}\left(\frac{4}{\sqrt{205}}\right)$$

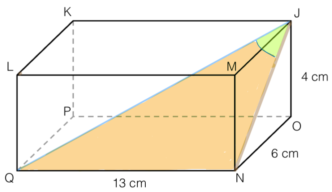
$$= \underline{15.6°}$$

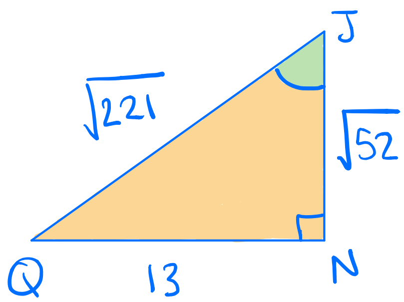
(b) Find the angle between JQ and the plane JKLM.



$$\angle QJL = \angle JQO = \underline{15.6°}$$

(c) Find the angle between JQ and the plane JMNO.





$$\angle QJN = \sin^{-1}\left(\frac{13}{\sqrt{221}}\right)$$

$$\text{or } \tan^{-1}\left(\frac{13}{\sqrt{52}}\right)$$

$$= \underline{61.0°}$$

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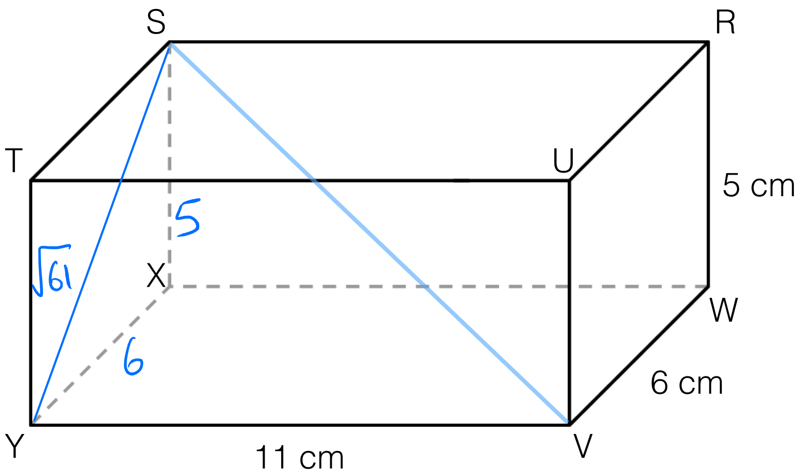
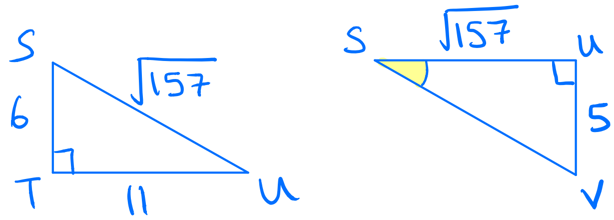
Alpha Exercise 2 ✓

Answer the following correct to the nearest 0.1°

$\sqrt{5^2+6^2} = \sqrt{61}$



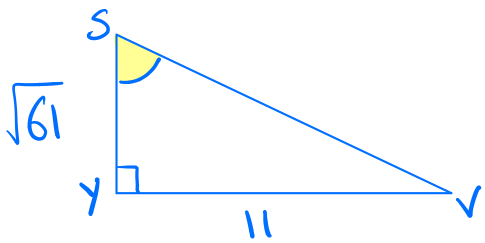
(a) Find the angle between SV and the plane RSTU.



$\angle XVS = \tan^{-1}\left(\frac{5}{\sqrt{157}}\right)$

$= \underline{21.8^\circ}$

(b) Find the angle between SV and the plane STYX.



$\angle VSY = \tan^{-1}\left(\frac{11}{\sqrt{61}}\right)$

$= \underline{54.6^\circ}$

(c) Find the angle between SV and the plane VWXY.

$\angle XVS = \angle VSU = \underline{21.8^\circ}$

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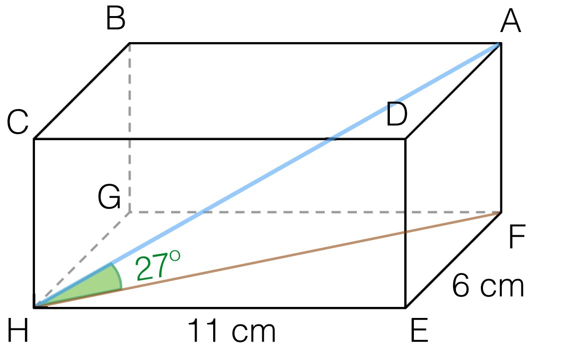
Beta Exercise

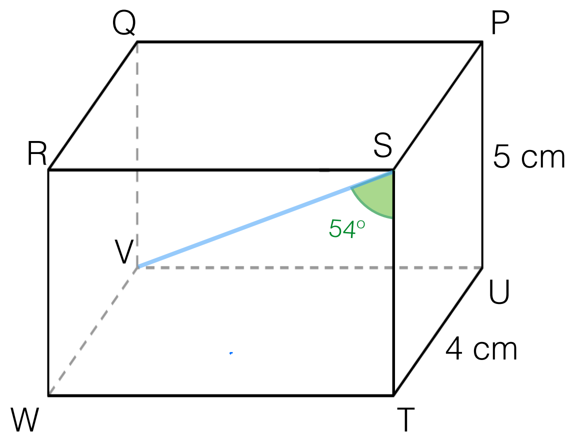
(a) Find the length AH, correct to the nearest 0.1 cm.

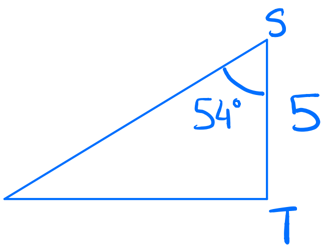


$$HF = \sqrt{6^2 + 11^2} = \sqrt{157}$$

$$AH = \frac{\sqrt{157}}{\cos(27)} = \underline{14.1\text{ cm}}$$

(b) Find the length SV, correct to the nearest 0.1 cm.

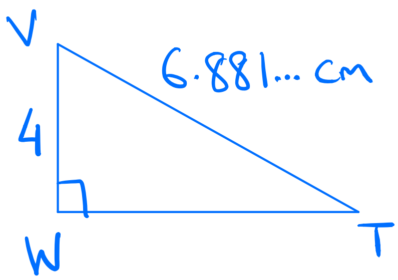




$$SV = \frac{5}{\cos(54)} = \underline{8.5\text{ cm}}$$

(c) Find the length TW, correct to the nearest 0.1 cm.

$$TV = 5\tan(54)$$
$$= 6.881\ldots\text{ cm}$$



$$TW = \sqrt{6.881\ldots^2 - 4^2} = \underline{5.6\text{ cm}}$$

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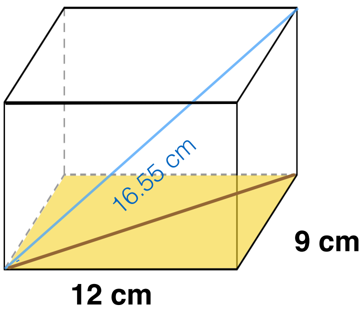
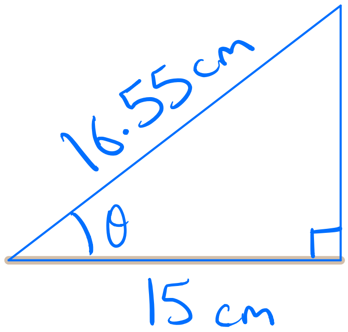
Gamma Exercise 1

Find the angle between each diagonal and the shaded plane

(a)



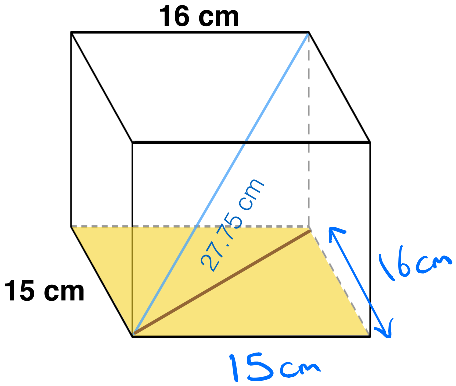


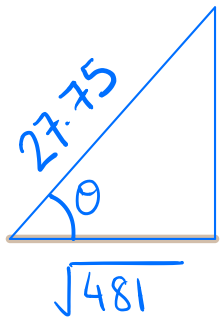
$$\sqrt{12^2+9^2}=15$$

$$\theta = \cos^{-1}\left(\frac{15}{16.55}\right)$$

$$= \underline{25.0^\circ}$$

(b)





$$\sqrt{15^2+16^2}=\sqrt{481}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{481}}{27.75}\right)$$

$$= \underline{37.8^\circ}$$

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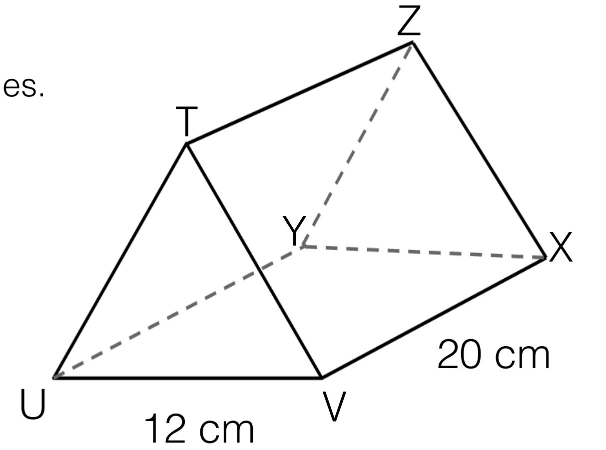
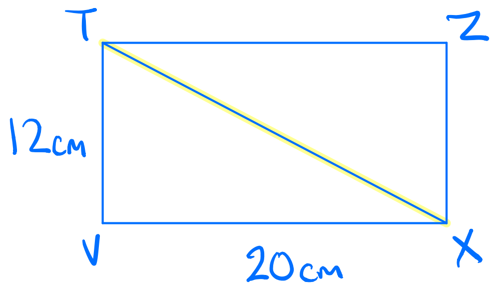
Gamma Exercise 2

Here is a **triangular prism**.
The triangular faces are **equilateral** triangles.

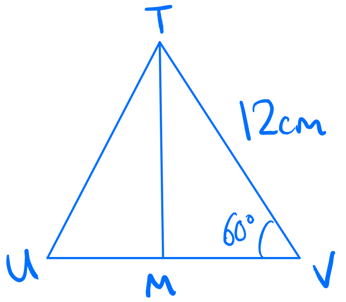


(a) Find the length TX



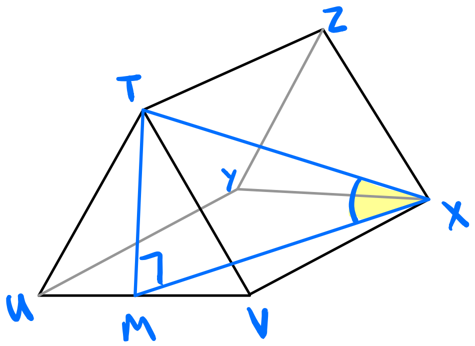
$TX = \sqrt{20^2 + 12^2} = \sqrt{544} = \underline{23.3}$ cm

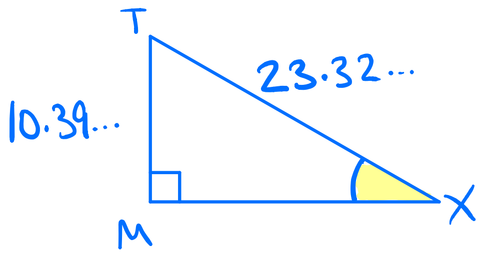
(b) M is the mid-point of UV. Find the length MT.



$MT = 12\sin(60°) = \underline{10.4}$ cm

(c) Find the angle between TX and the plane UVXY.





$\angle TXM = \sin^{-1}\left(\frac{10.39...}{23.32...}\right)$

$= \underline{26.5°}$

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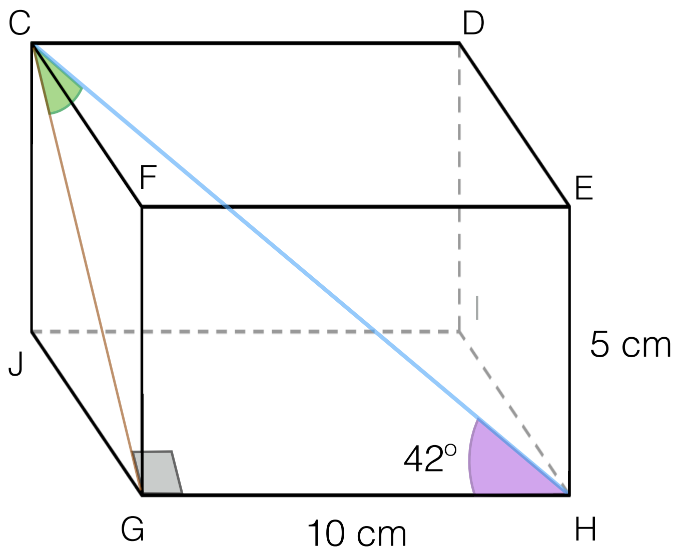
Explain the mistake

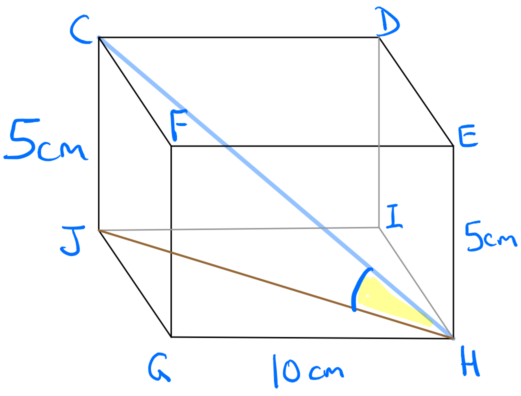


Molly writes:

The angle between CH and the plane GHIJ is 42°, and the angle between CH and the plane CJGF is 48°.

Molly has made a mistake. Identify the mistake and correct it.

The angle between CH and CJGF is 48°, but the angle between CH and GHIJ is not 42°.



$$\text{Length } CH = \frac{10}{\cos(42)} = 13.45\ldots \text{ cm}$$

$$\angle CHJ = \sin^{-1}\left(\frac{5}{13.45\ldots}\right)$$

$$=$$

The angle between CH and GHIJ is $\underline{21.8^\circ}$.

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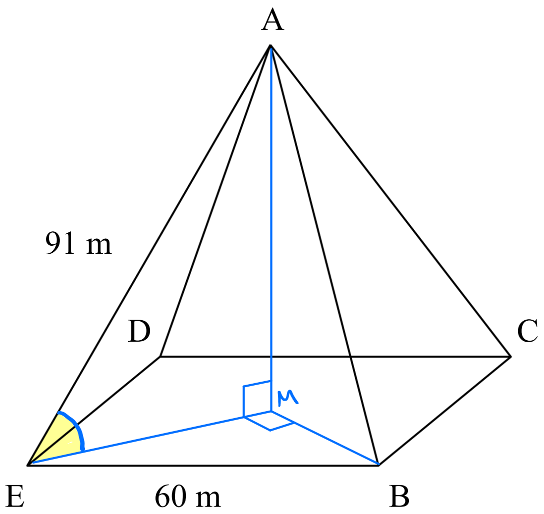
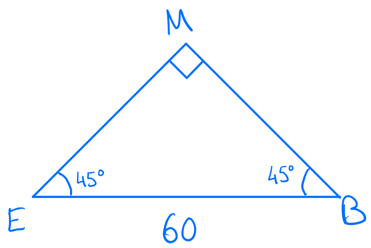
Exam-style question

This pyramid has a square base of length 60 m. The length from a corner of the base to the top of the pyramid is 91 m.

Find the angle between AB and the plane BCDE.

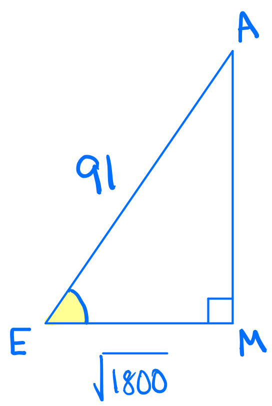




Right-angled isosceles

$$EM = \sqrt{\frac{60^2}{2}}$$

$$= \sqrt{1800} \text{ m}$$



$$\angle AEM = \cos^{-1}\left(\frac{\sqrt{1800}}{91}\right)$$

$$= \underline{62.2^\circ}$$