Name: Solutions

Practice Paper for AQA Level 2 Certificate
FURTHER MATHEMATICS
Paper 1 Non-Calculator

Time allowed: 1 hour 45 minutes

Materials
For this paper you must have:
- mathematical instruments.

You must **not** use a calculator.

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets
- The maximum mark for this paper is 80.
- You may ask for more graph paper and tracing paper. These must be tagged securely to this answer book.

Copies of this paper and worked solutions can be found at bossmaths.com/level2fmpractice, also accessible via this QR code.

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Answer **all** questions in the spaces provided.

1 Factorise fully $6a^4b - 15ab^3$

[2 marks]

Answer $3ab(2a^3 - 5b^2)$

2 Work out the values of p, q and r such that $2x^2 - 8x + p \equiv q(x + r)^2 + 19$

[3 marks]

$\underline{q = 2}$, so $2x^2 - 8x + p = 2(x + r)^2 + 19$

$\Rightarrow 2x^2 - 8x + p = 2(x^2 + 2rx + r^2) + 19$

$\Rightarrow 2x^2 - 8x + p = 2x^2 + 4rx + 2r^2 + 19$

Equating coefficients: $4r = -8 \Rightarrow \underline{r = -2}$

$\underline{p = 2r^2 + 19 = 2(-2)^2 + 19 = 27}$

$p =$ 27 $q =$ 2 $r =$ -2

3 Work out $\begin{pmatrix} 2 & 6 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 8 & 1 \\ 10 & 3 \end{pmatrix}$

[2 marks]

$$\begin{array}{c|c} & \begin{pmatrix} 8 & 1 \\ 10 & 3 \end{pmatrix} \\ \hline \begin{pmatrix} 2 & 6 \\ 4 & 5 \end{pmatrix} & \begin{pmatrix} 76 & 20 \\ 82 & 19 \end{pmatrix} \end{array}$$

Answer $\begin{pmatrix} 76 & 20 \\ 82 & 19 \end{pmatrix}$

$5x^4 + 12x^2 - 12x - 5$

4 How many numbers satisfy **all** of the following conditions?

$20x^3 + 24x - 5$

- The number is a four-digit integer..
- The second digit is 8.
- The other digits are all odd numbers.

$1, \quad \frac{d^2y}{dx^2} = 20 + 24 - 5 = 39$

[3 marks]

$\boxed{}\ \boxed{8}\ \boxed{}\ \boxed{}$ i.e. $\frac{d^2y}{dx^2} > 0$

Minimum

Five choices available for each of these three:

1, 3, 5, 7, 9

∴ there are $5 \times 5 \times 5 = 125$ numbers that satisfy all the conditions.

$2g(x) - 1$

$2(x^2 + 3) - 1$

Answer 125

$= 2x^2 + 6 - 1$

$2x^2 + 5$

5 Simplify $\sqrt{8}(\sqrt{98} + \sqrt{32} - \sqrt{50})$ writing your answer as an integer.

[3 marks]

$2\sqrt{2}\,(7\sqrt{2} + 4\sqrt{2} - 5\sqrt{2})$

$= 2\sqrt{2}\,(6\sqrt{2})$

$= 2 \times 6 \times \sqrt{2} \times \sqrt{2} = 12 \times 2 = 24$

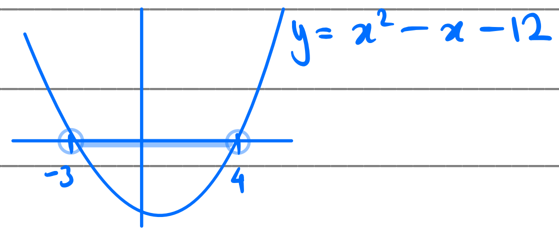
Answer 24

6 Solve $x^2 - x < 12$

[2 marks]

$x^2 - x - 12 < 0$

$\Rightarrow (x-4)(x+3) < 0$



Answer $-3 < x < 4$

7 Use **matrix multiplication** to show that, in the x-y plane,

- a reflection in the line $y = x$, followed by
- a reflection in the line $y = -x$

is equivalent to a rotation of 180º about the origin.

[3 marks]

Reflection in $y=x$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Reflection in $y=-x$ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

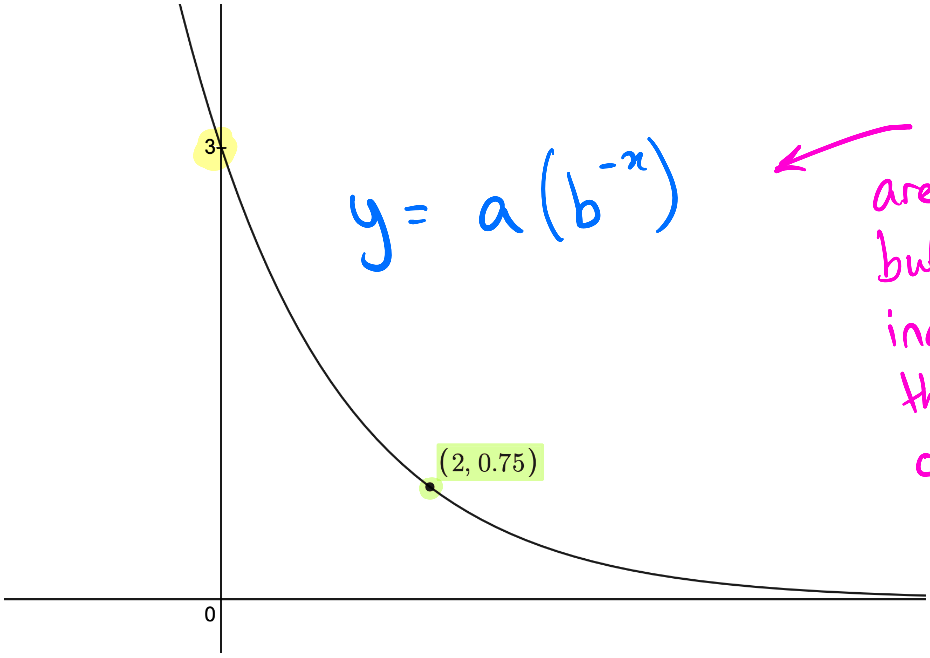
$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},$$

which is the matrix for a 180° rotation about the origin.

NOTE THE CORRECT ORDER OF THE MATRIX MULTIPLICATION!
(In this particular question, swapping the order will still give the same result, but make sure to get the order correct to get all the marks.)

8 The curve shown has equation $y = ab^{-x}$ where a and b are positive integers. The point $(2, 0.75)$ lies on the curve. Find the values of a and b.

[2 marks]



$y = a(b^{-x})$

The brackets are not necessary but have been included to make the order of operations clear.

$(0,3)$ lies on the curve so $3 = a(b^0)$

i.e. $\underline{3 = a}$

$(2, 0.75)$ lies on the curve so

$0.75 = 3(b^{-2})$

$\Rightarrow \quad 0.25 = b^{-2} \quad \Rightarrow \quad \frac{1}{4} = \frac{1}{b^2}$

$\Rightarrow \quad \underline{b = 2}$

$a =$ 3 $\quad b =$ 2

9 The curve with equation $y = x^5 + 4x^3 - 6x^2 - 5x + 13$ has a turning point at $(1,7)$.
Determine whether this point is a maximum or a minimum.

[2 marks]

$$\frac{dy}{dx} = 5x^4 + 12x^2 - 12x - 5$$

$$\frac{d^2y}{dx^2} = 20x^3 + 24x - 12$$

When $x = 1$, $\frac{d^2y}{dx^2} = 20 + 24 - 12 = 32$

i.e. $\frac{d^2y}{dx^2} > 0$

Answer Minimum

10 $f(x) = 2x - 1$
$g(x) = x^2 + 3$

Work out $fg(x)$

[2 marks]

$$f(g(x)) = 2g(x) - 1$$

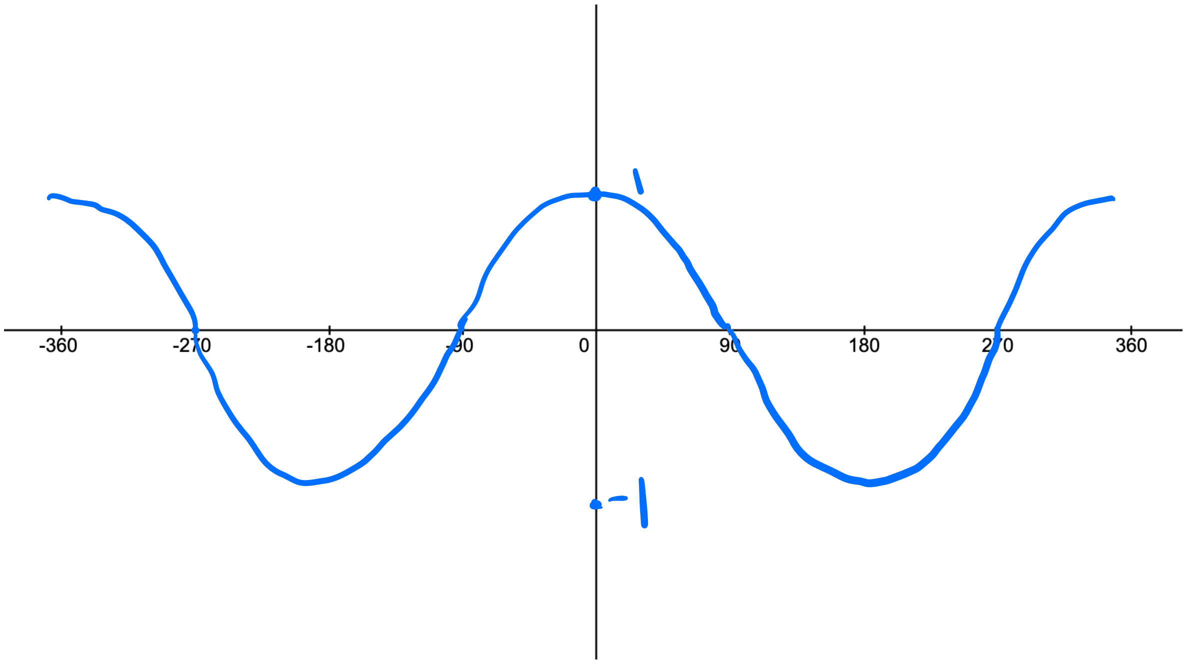
$$= 2(x^2 + 3) - 1$$

$$= 2x^2 + 6 - 1$$

= Answer $2x^2 + 5$

11 **(a)** On the axes, sketch $y = \cos x$ for $-360° \leqslant x \leqslant 360°$

[3 marks]



(b) You are given that $180° < u < 360°$ and that $\tan u = 1$
Find the value of u.

[1 mark]

tan u = 1 when u = 45° ± multiples of 180°

Answer 225°

12 The nth term of a sequence is U_n

$$U_n = \frac{4n - 11}{5n}$$

(a) Work out the least value of n for which $U_n \geq 0.7$

[3 marks]

$$\frac{4n-11}{5n} \geqslant 0.7$$

$$\Rightarrow 4n - 11 \geqslant 3.5n$$

$$\Rightarrow 0.5n \geqslant 11$$

$$\Rightarrow n \geqslant 22$$

Answer 22

(b) Write down the limiting value of U_n as $n \to \infty$

[1 mark]

Answer $\frac{4}{5}$ or 0.8

13 **(a)** Show that $(2\sin\theta + \cos\theta)(\cos\theta - 2\sin\theta) + 2 \equiv 5\cos^2\theta - 2$

Difference of two squares

[3 marks]

$(2\sin\theta + \cos\theta)(\cos\theta - 2\sin\theta) + 2$

$\equiv \cos^2\theta - 4\sin^2\theta + 2$

$\equiv \cos^2\theta - 4(1 - \cos^2\theta) + 2$

$\equiv \cos^2\theta - 4 + 4\cos^2\theta + 2$

$\equiv 5\cos^2\theta - 2$ as required.

(b) Hence, or otherwise, find the value of $(2\sin 60° + \cos 60°)(\cos 60° - 2\sin 60°)$

[3 marks]

Subtracting 2 from each side of the given identity and substituting $\theta = 60$:

$(2\sin 60 + \cos 60)(\cos 60 - 2\sin 60)$

$= 5\cos^2 60 - 4 \quad = 5\left(\frac{1}{2}\right)^2 - 4$

$= \frac{5}{4} - \frac{16}{4}$

Answer $-\frac{11}{4}$

14 The table lists the equations of three straight lines.

	Equation of line	Gradient	y-intercept
1	$y = 7 - 3x$	-3	7
2	$5x + 2y = 20$	$-\frac{5}{2}$	10
3	$\frac{y - 21}{x - 5} = 3$	3	6

Fill in the gradients and y-intercepts of each line.

[6 marks]

Workings for 2 and 3

2 $5x + 2y = 20$

$\Rightarrow \quad 2y = -5x + 20$

$\Rightarrow \quad y = -\frac{5}{2}x + 10$

3 $\frac{y-21}{x-5} = 3$

Note this is in the form $\frac{y-y_1}{x-x_1} = m$

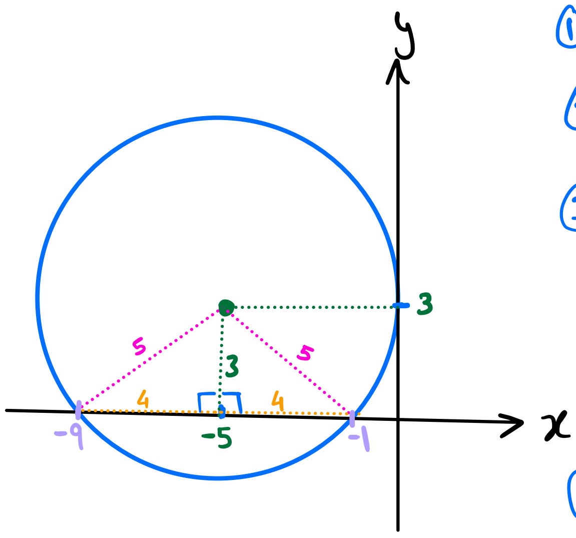
$\Rightarrow \quad y - 21 = 3(x-5)$

$\Rightarrow \quad y - 21 = 3x - 15$

$\Rightarrow \quad y = 3x + 6$

15 **(a)** The circle with equation $(x+5)^2 + (y-3)^2 = 25$ has three points of intersection with the coordinate axes. Find the coordinates of the these three points.

[3 marks]



① Centre $(-5,3)$

② Radius 5

③ By Pythagoras, the two right-angled have a third side length of 4.

④ Hence the circle intersects at -1 and 9 on the x-axis.

Answer $(0,3)$, $(-1,0)$, $(-9,0)$

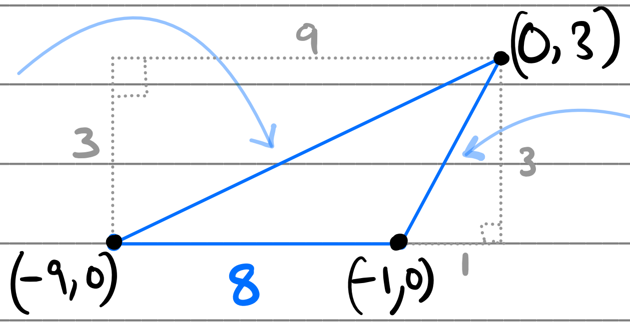
(b) The three points of intersection are joined to form a triangle. Work out the ratio of the length of this triangle's shortest side to its longest side. Write your answer in the form 1: n, where n is an integer.

[4 marks]

$\sqrt{9^2+3^2}$

$= \sqrt{90}$

(longest)



$\sqrt{1^2+3^2} = \sqrt{10}$

(shortest)

$\sqrt{10} : \sqrt{90} = \sqrt{10} : \sqrt{9}\sqrt{10} = \sqrt{10} : 3\sqrt{10}$

Answer 1 : 3

16 **(a)** The point P has coordinates $(2, a)$. P lies on the curve with equation $y = x^3 + 5x^2 - 4x - 20$. Find the value of a.

[1 mark]

$a = 2^3 + 5(2^2) - 4(2) - 20$

$= 8 + 20 - 8 - 20 = 0$

Answer 0

(b) Find the equation of the normal to the curve at point P, writing your answer in the form $y = mx + c$

[4 marks]

$\frac{dy}{dx} = 3x^2 + 10x - 4$

At $(2,0)$, gradient of curve $= 3(2^2) + 10(2) - 4$

$= 12 + 20 - 4 = 28$

$\therefore$ Gradient of normal $= -\frac{1}{28}$

Equation of normal: $y - 0 = -\frac{1}{28}(x - 2)$

Answer $y = -\frac{1}{28}x + \frac{1}{14}$

17 $f(x) = x^3 + ax^2 + bx + c$

You are given that $x^2 - x - 12$ is a factor of $f(x)$.
You are also given that $x^2 - 9x + 20$ is a factor of $f(x)$.

Find a, b, and c.

[5 marks]

$x^2 - x - 12 \equiv (x-4)(x+3)$

i.e. $(x-4)$ and $(x+3)$ are factors of $f(x)$.

$x^2 - 9x + 20 \equiv (x-4)(x-5)$

i.e. $(x-4)$ and $(x-5)$ are factors of $f(x)$.

$f(x) \equiv (x-4)(x+3)(x-5)$

$\equiv (x^2 - x - 12)(x-5)$

$\equiv x^3 - x^2 - 12x - 5x^2 + 5x + 60$

$\equiv x^3 - 6x^2 - 7x + 60$

$a =$ -6 $b =$ -7 $c =$ 60

18 Solve the simultaneous equations.

$$3x + 3y + 6z = 9 \quad ①$$
$$x - y + 2z = -1 \quad ②$$
$$5x - 2z = 2 \quad ③$$

Do **not** use trial and improvement.
You **must** show your working.

[5 marks]

① ÷ 3 $\quad x + y + 2z = 3 \quad$ ④

② $\quad x - y + 2z = -1$

Adding $\quad 2x + 4z = 2$

÷ 2 $\quad x + 2z = 1 \quad$ ⑤

③ $\quad 5x - 2z = 2$

Adding $\quad 6x = 3$

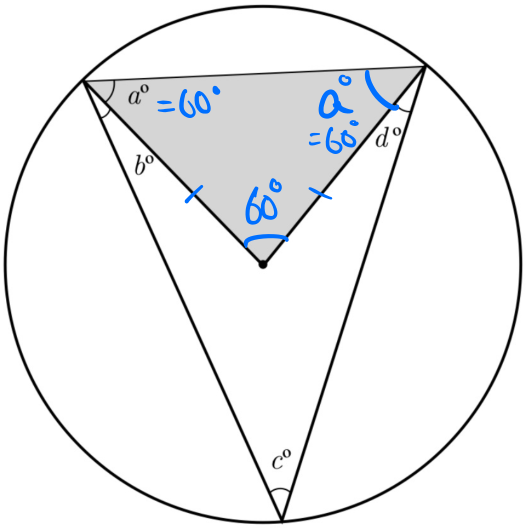
$\Rightarrow \quad x = \frac{1}{2}$

Sub into ⑤ to find $z = \frac{1}{4}$

Sub both into ④ to find $y = 2$

$x = \frac{1}{2}$ $\quad y = 2$ $\quad z = \frac{1}{4}$

19 This circle has radius 7 cm.
You are given that $a : b : c : d = 6 : 2 : 3 : 1$



i.e. $a = 6d$
$b = 2d$
$c = 3d$

Diagram not drawn accurately.

Find the shaded area in cm^2, giving your answer in the form $\frac{u\sqrt{v}}{w}$, where u, v, and w are integers.

[5 marks]

$2a + b + c + d = 180$

$\Rightarrow 12d + 2d + 3d + d = 180$

$\Rightarrow 18d = 180 \Rightarrow d = 10 \Rightarrow a = 60°$

$\therefore$ Shaded triangle is equilateral.

Area $= \frac{1}{2} \times 7 \times 7 \sin(60)$

$= \frac{49}{2} \times \frac{\sqrt{3}}{2} = \frac{49\sqrt{3}}{4}$

Answer $\frac{49\sqrt{3}}{4}$

20 You are given that $10 \times 5^{17} + 23 \times 5^{18} = 5^n$
Find the value of n.

[4 marks]

$10 \times 5^{17} + 23 \times 5^{18}$

$= 2 \times 5 \times 5^{17} + 23 \times 5^{18}$

$= 2 \times 5^{18} + 23 \times 5^{18}$

$= 25 \times 5^{18}$

$= 5^2 \times 5^{18}$

$= 5^{20}$

$n =$ 20

21 Prove that the difference between the squares of two different odd numbers is always a multiple of 8.

[5 marks]

Let m, n be positive integers, with $m > n$.
Then $2m+1$ and $2n+1$ are odd

$(2m+1)^2 - (2n+1)^2$

$\equiv (4m^2 + 4m + 1) - (4n^2 + 4n + 1)$

$\equiv 4m^2 + 4m - 4n^2 - 4n$

$\equiv 4(m^2 - n^2 + m - n)$

$\equiv 4((m+n)(m-n) + (m-n))$

$\equiv 4(m+n+1)(m-n)$

If m and n are both even or both odd, then $(m-n)$ is even.
Otherwise $(m+n+1)$ is even.
i.e. either $(m+n+1)$ or $(m-n)$ is even

$\therefore$ $4(m+n+1)(m-n)$ is a product of 4 and another even number so a multiple of 8.