Name: Solutions

Practice Paper for AQA Level 2 Certificate
FURTHER MATHEMATICS
Paper 2 Calculator

Time allowed: 1 hour 45 minutes

Materials
For this paper you must have:
- a calculator
- mathematical instruments.

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets
- The maximum mark for this paper is 80.
- You may ask for more graph paper and tracing paper. These must be tagged securely to this answer book.

Copies of this paper and worked solutions can be found at bossmaths.com/level2fmpractice, also accessible via this QR code.

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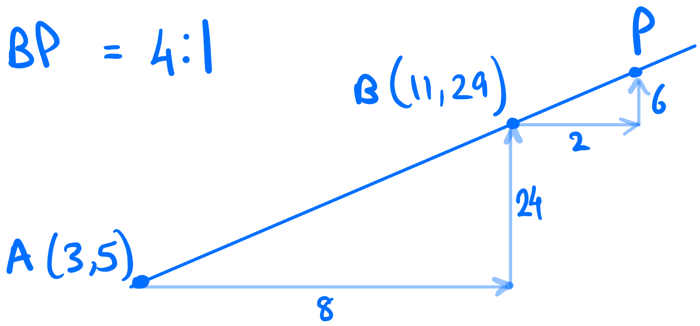
Answer **all** questions in the spaces provided.

1 A has coordinates $(3,5)$ and B has coordinates $(11,29)$. P lies on the line through A and B such that the ratio of the distance AB to the distance AP is $4:5$. Find the coordinates of P.

$AB : AP = 4:5$

[3 marks]

so $AB : BP = 4:1$



Answer $(13, 35)$

2 The first terms of a linear sequence are:

$$9a - 2b, 5a + b, a + 4b, \ldots$$

Work out an expression for the nth term of this sequence.

[3 marks]

$-4a + 3b \qquad -8a + 6b \qquad -12 + 9b$

$\downarrow +13a - 5b \qquad \downarrow +13a - 5b \qquad \downarrow +13a - 5b$

$9a - 2b \qquad 5a + b \qquad a + 4b$

$-4a + 3b \qquad -4a + 3b$

Answer $(-4a + 3b)n + 13a - 5b$

3 $\mathrm{f}(x) = \dfrac{5x-4}{7x+9}$

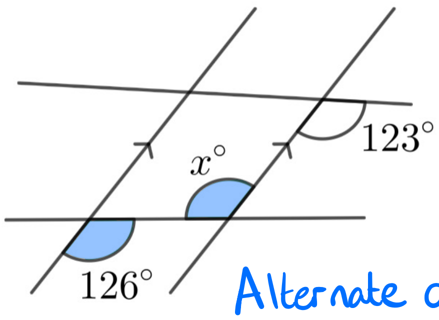
Which value of x can **not** be in the domain of $\mathrm{f}(x)$?
Circle your answer.

[1 mark]

$-\frac{9}{7}$ $\quad$ $-\frac{7}{9}$ $\quad$ 0 $\quad$ $\frac{7}{9}$ $\quad$ $\frac{4}{5}$

4 Write down the value of x.

[1 mark]



Alternate angles

$x =$ 126

5 $f(x) = 3x^3 - 4x + 25$

Find the set of values of x for which $f(x)$ is decreasing. → gradient is negative.

[3 marks]

Say $y = f(x)$.

Then $\frac{dy}{dx} = 9x^2 - 4$

$\frac{dy}{dx} < 0 \Rightarrow 9x^2 - 4 < 0$

$\Rightarrow (3x+2)(3x-2) < 0$

Answer $-\frac{2}{3} < x < \frac{2}{3}$

6 Solve $\frac{6}{x} = 1 + \frac{1}{x^2}$, giving your solutions to 3 significant figures.

[2 marks]

$6x = x^2 + 1$

$\Rightarrow 0 = x^2 - 6x + 1$

$\Rightarrow 0 = (x-3)^2 - 8$

$\Rightarrow 8 = (x-3)^2$

$\Rightarrow \pm\sqrt{8} = x - 3$

$\Rightarrow 3 \pm \sqrt{8} = x$

Answer $x = 0.172$, $x = 2.83$

7 Use **matrix multiplication** to show that, in the $x-y$ plane,

- a reflection in the line y-axis, followed by
- a rotation by 90º anti-clockwise about (0,0) is equivalent

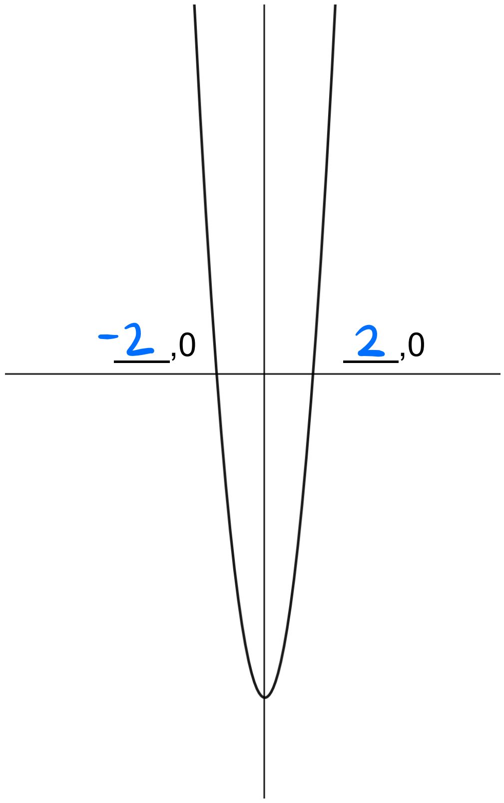
is equivalent to a reflection in the line $y = -x$

[3 marks]

$$\underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}_{\text{Rotation 90° anticlockwise about (0,0)}} \underbrace{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}_{\text{Reflection in y-axis}} = \underbrace{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}}_{\text{Reflection in } y=-x}$$

8 **(a)** The curve shown has equation $y = 3x^2 - 12$.
Fill in the x-coordinates of the points where the curve intersects the x-axis.

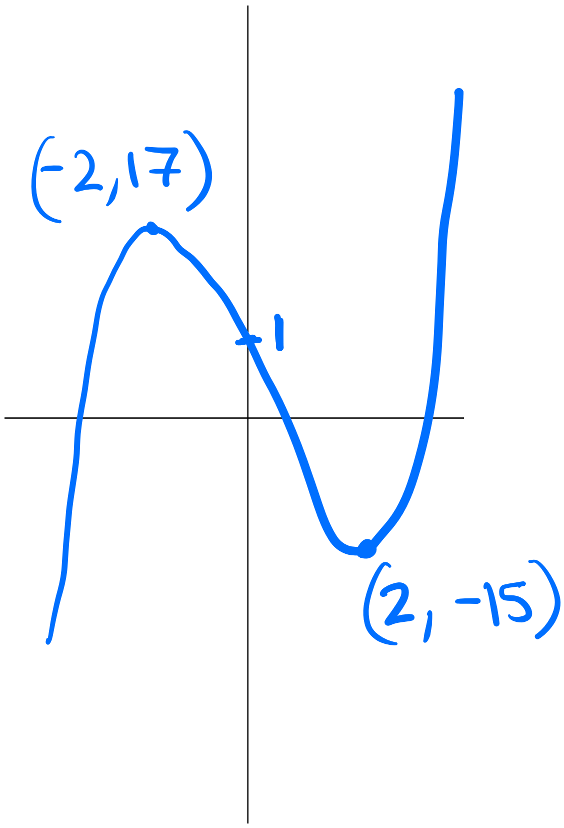
[2 marks]



$$y = 3(x^2 - 4)$$
$$= 3(x+2)(x-2)$$

(b) Hence, or otherwise, sketch the curve with equation $y = x^3 - 12x + 1$.
Clearly show any stationary points and label the coordinates of these points.

[3 marks]



$$\frac{dy}{dx} = 3x^2 - 12$$

$\Rightarrow$ stationary points at $x = 2$ and $x = -2$

9 Show that $\frac{\cos^2\theta+\tan\theta+\sin^2\theta}{\sin\theta} \equiv \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$

[2 marks]

$$\frac{\cos^2\theta + \tan\theta + \sin^2\theta}{\sin\theta} = \frac{\tan\theta + 1}{\sin\theta}$$

$$= \frac{\tan\theta}{\sin\theta} + \frac{1}{\sin\theta} = \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\sin\theta} + \frac{1}{\sin\theta}$$

$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$

10 Rearrange $v = \sqrt{\frac{3+4w}{u+1}}$ to make u the subject

[3 marks]

$$v^2 = \frac{3+4w}{u+1}$$

$$\Rightarrow u+1 = \frac{3+4w}{v^2}$$

$$\Rightarrow u = \frac{3+4w}{v^2} - 1$$

or $\frac{3+4w-v^2}{v^2}$

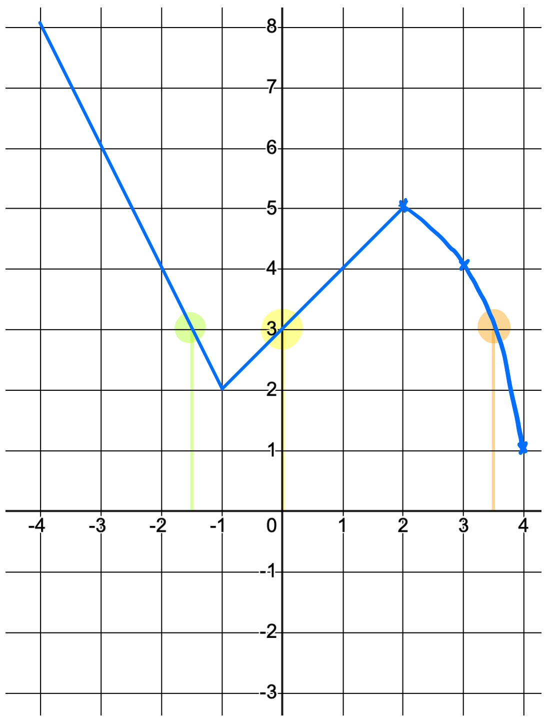
Answer $u = \frac{3+4w}{v^2} - 1$

11 **(a)** A function f is given by

$$f(x) = \begin{cases} -2x & x < -1 \\ x + 3 & -1 \le x < 2 \\ -x^2 + 4x + 1 & x \ge 2 \end{cases}$$

Plot $y = \mathrm{f}(x)$ on the axes given.

[3 marks]



(b) Work out **all** the values of x for which $\mathrm{f}(x) = 3$.

[4 marks]

① $-2x = 3 \Rightarrow x = -\frac{3}{2}$

② $x + 3 = 3 \Rightarrow x = 0$

③ $-x^2 + 4x + 1 = 3 \Rightarrow 0 = x^2 - 4x + 2$

$\Rightarrow x = 2 \pm \sqrt{2}$

The solution between 3 and 4 is $2 + \sqrt{2}$

≈ 3.41

Answer $x = -\frac{3}{2}$, $x = 0$, $x = 2 + \sqrt{2}$

12 Given that $(x+3)$ is a factor of $x^3 - 2x^2 - 2px + 3p^2$ where p is a constant, find the two possible values of p.

[4 marks]

$(x+3)$ is a factor

$\Rightarrow (-3)^3 - 2(-3)^2 - 2p(-3) + 3p^2 = 0$

$\Rightarrow -27 - 18 + 6p + 3p^2 = 0$

$\Rightarrow 3p^2 + 6p - 45 = 0$

$\Rightarrow p^2 + 2p - 15 = 0$

$\Rightarrow (p+5)(p-3) = 0$

$p =$ -5 , $p =$ 3

13 $f(x) = 2x + 3$

Solve $f^{-1}(2k) = \frac{k}{5}$

[4 marks]

$f^{-1}(x) = \frac{x-3}{2}$

So $f^{-1}(2k) = \frac{2k-3}{2}$

Solving. $\frac{2k-3}{2} = \frac{k}{5}$

$\Rightarrow 5(2k-3) = 2k$

$\Rightarrow 10k - 15 = 2k$

$\Rightarrow 8k = 15$

$\Rightarrow k = \frac{15}{8}$

Answer $k = \frac{15}{8}$

14 The following curve and straight line intersect at two points. Find the midpoint of these two points of intersection.

Method ①

$$x^2 + 7xy + 4y^2 - 256 = 0$$
$$x - y - 8 = 0 \Rightarrow x = y + 8$$

Do **not** use trial and improvement.
You **must** show your working.

[6 marks]

$(y+8)^2 + 7y(y+8) + 4y^2 - 256 = 0$

$\Rightarrow y^2 + 16y + 64 + 7y^2 + 56y + 4y^2 - 256 = 0$

$\Rightarrow 12y^2 + 72y - 192 = 0$

$\Rightarrow y^2 + 6y - 16 = 0$

$\Rightarrow (y+8)(y-2) = 0$

$\Rightarrow y = -8 \qquad y = 2$

$x = 0 \qquad x = 10$ } since $x = y+8$

Midpoint of $(0,-8)$ and $(10,2)$

is $(5,-3)$

Answer $(5,-3)$

14 The following curve and straight line intersect at two points. Find the midpoint of these two points of intersection.

Method ②

$$x^2 + 7xy + 4y^2 - 256 = 0$$
$$x - y - 8 = 0 \Rightarrow y = x - 8$$

Do **not** use trial and improvement.
You **must** show your working.

[6 marks]

$x^2 + 7x(x-8) + 4(x-8)^2 - 256 = 0$

$\Rightarrow x^2 + 7x^2 - 56x + 4(x^2 - 16x + 64) - 256 = 0$

$\Rightarrow 8x^2 - 56x + 4x^2 - 64x + 256 - 256 = 0$

$\Rightarrow 12x^2 - 120x = 0$

$\Rightarrow 12x(x-10) = 0$

$\Rightarrow x = 0 \qquad x = 10$

$y = -8 \qquad y = 2$ } since $y = x - 8$

Midpoint of $(0,-8)$ and $(10,2)$

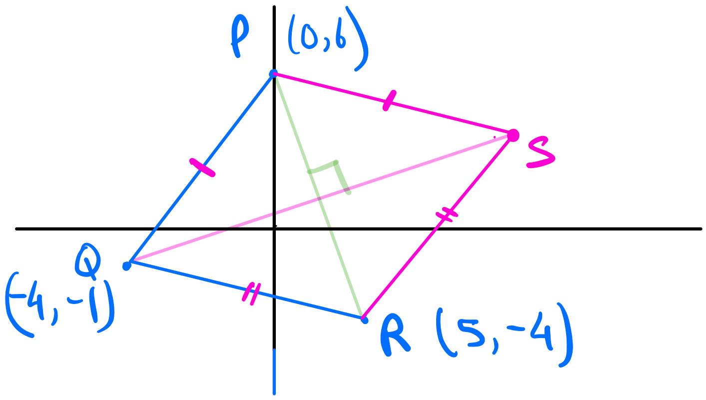
is $(5,-3)$

Answer $(5,-3)$

15 *PQRS* is a kite.
P has coordinates $(0,6)$.
Q has coordinates $(-4,-1)$.
R has coordinates $(5,-4)$.

Find the equation of the straight line that passes through Q and S, giving your answer in the form $ax + by + c = 0$.

[4 marks]



PR has gradient $\frac{-4-6}{5-0} = \frac{-10}{5} = -2$

Diagonals of a kite are perpendicular.

So QS has gradient $\frac{1}{2}$.

Line through QS: $y - -1 = \frac{1}{2}(x - -4)$

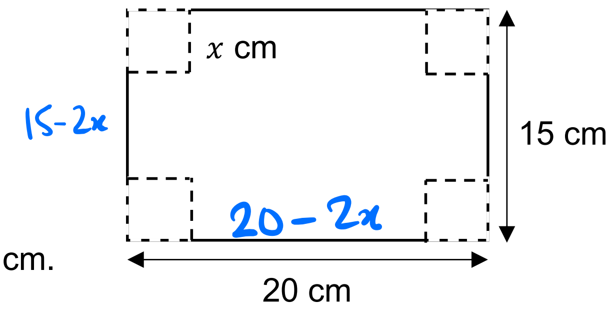
$\Rightarrow y+1 = \frac{1}{2}x + 2$

$\Rightarrow 2y+2 = x+4$

Answer $-x + 2y - 2 = 0$

or $x - 2y + 2 = 0$

16 Squares of side length x cm are cut from a 20 cm by 15 cm rectangular piece of cardboard. The carboard is then folded to create an open-topped box.



(a) Explain why x must be less than 7.5 cm.

[1 mark]

The shorter side of the cardboard is 15 cm.

(b) Show that $V = 4x^3 - 70x^2 + 300x$, for $x < 7.5$, where V is the volume of the box in cm^3.

[2 marks]

$V = x(15-2x)(20-2x)$

$= x(300 - 40x - 30x + 4x^2)$

$= 4x^3 - 70x^2 + 300x$

(c) Use calculus to work out the maximum possible volume of the box, giving your answer correct to 3 significant figures.

[4 marks]

$\frac{dV}{dx} = 12x^2 - 140x + 300$

$\frac{dV}{dx} = 0$ when $12x^2 - 140x + 300 = 0$

Solutions $x = 2.829\ldots$, ~~$x = 8.838\ldots$~~

$x \geq 7.5$, so reject

$V = 379 \text{ cm}^3$

Answer 379 cm^3

17 The first term of a quadratic sequence is 4.
The second term of this sequence is 3.
The fourth term of this sequence is 7.
The fifth term of this sequence is 12.

Find an expression for the nth term of this sequence.

[6 marks]

Method ①

4 3 4 7 12

−1 1 3 5 1st differences

2 2 2 2nd differences

The 1st differences must form a linear sequence, so the blanks must be 1 and 3, so the third term of the quadratic sequence must be 4.

The second difference is therefore 2, so the coefficient of n^2 is half of 2 i.e. 1.

$1n^2$	1	4	9	16	25
$-4n+7$	3	−1	−5	−9	−13
n^2-4n+7	4	3	4	7	12

Answer $n^2 - 4n + 7$

17 The first term of a quadratic sequence is 4.
The second term of this sequence is 3.
The fourth term of this sequence is 7.
The fifth term of this sequence is 12.

Method 2

Find an expression for the nth term of this sequence.

[6 marks]

n^{th} term $= an^2 + bn + c$

1st term $a + b + c = 4$ (1)

2nd term $4a + 2b + c = 3$ (2)

4th term $16a + 4b + c = 7$ (3)

(2) − (1) $3a + b = -1$ (4)

(5) − (2) $12a + 2b = 4$ (5)

2 × (4) $6a + 2b = -2$ (6)

(5) − (6) $6a = 6$

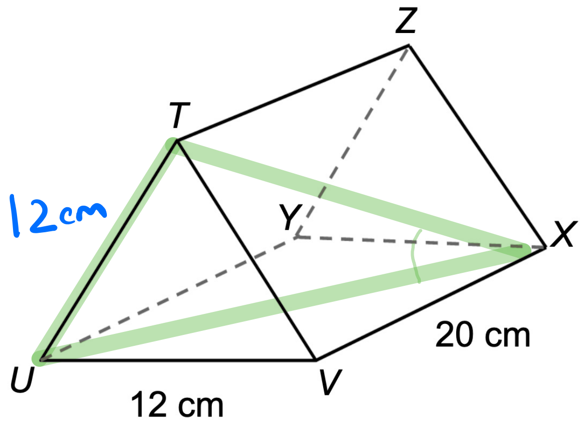
$\Rightarrow a = 1$

$\Rightarrow b = -4$ sub $a=1$ into (4) or (5)

$\Rightarrow c = 7$ sub $a=1, b=-4$ into (1), (2), or (3)

Answer $n^2 - 4n + 7$

18 Here is a triangular prism. The triangular faces are equilateral.
UV has length 12 cm and *VX* has length 20 cm.
Work out the size of angle *TXU*, giving your answer to the nearest 0.1°.

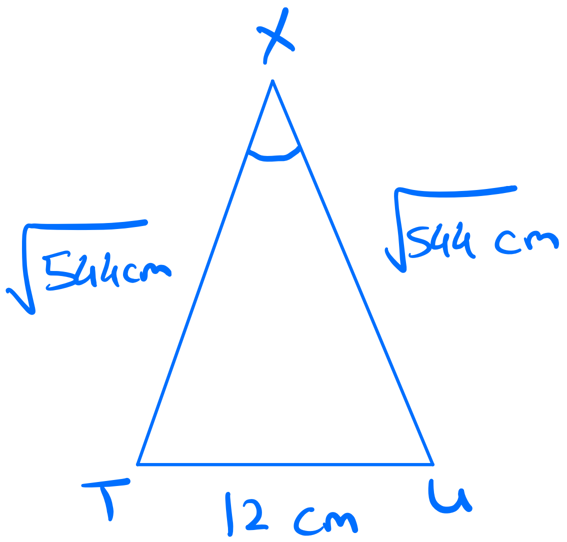


$TX = UX$

$= \sqrt{12^2 + 20^2}$

$= \sqrt{544}$

[5 marks]



$\cos(\angle TXU)$

$= \dfrac{544 + 544 - 144}{2 \times \sqrt{544} \times \sqrt{544}}$

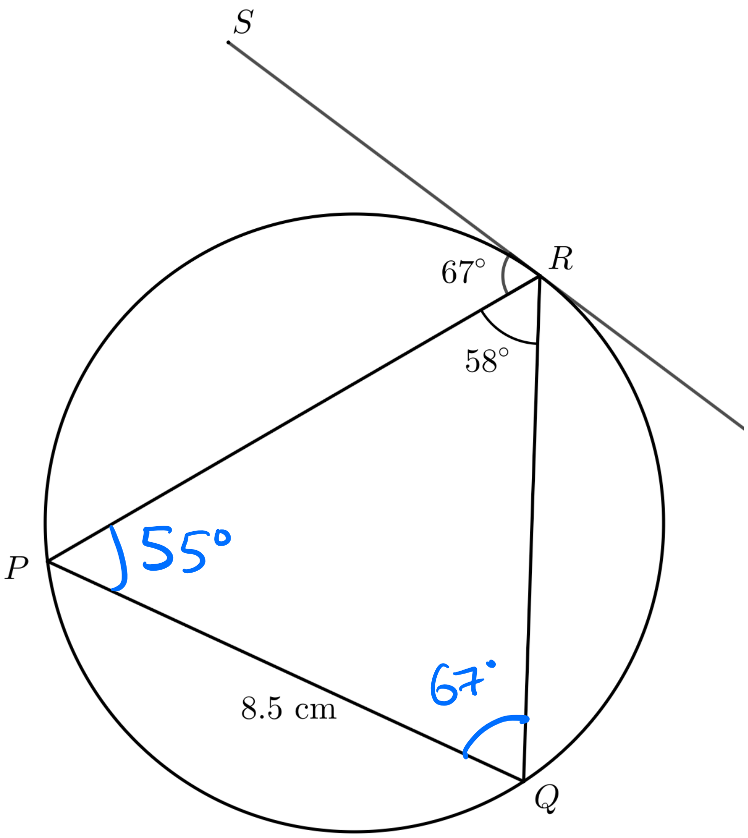
$= \dfrac{944}{1088}$

$\angle TXU = \cos^{-1}\left(\dfrac{944}{1088}\right)$

$= 29.8^\circ$

Answer ____________

19 The diagram shows a circle with points *P*, *Q*, and *R* on its circumference. The line shown is tangent to the circle at the point *R*.
Angle SRP = $67°$
Angle *PRQ* = $58°$
Chord *PQ* has length 8.5 cm.
Find the length of chord *QR*, giving your answer correct to 3 significant figures.



Not drawn accurately.

[4 marks]

$\angle RQP = 67°$ (alternate segment theorem)

$\angle QPR = 65°$ (angles in a triangle sum to 180°)

$$\frac{QR}{\sin 55} = \frac{8.5}{\sin 58}$$

$$\Rightarrow QR = \frac{8.5 \sin 55}{\sin 58} = 8.21$$

Answer 8.21 cm

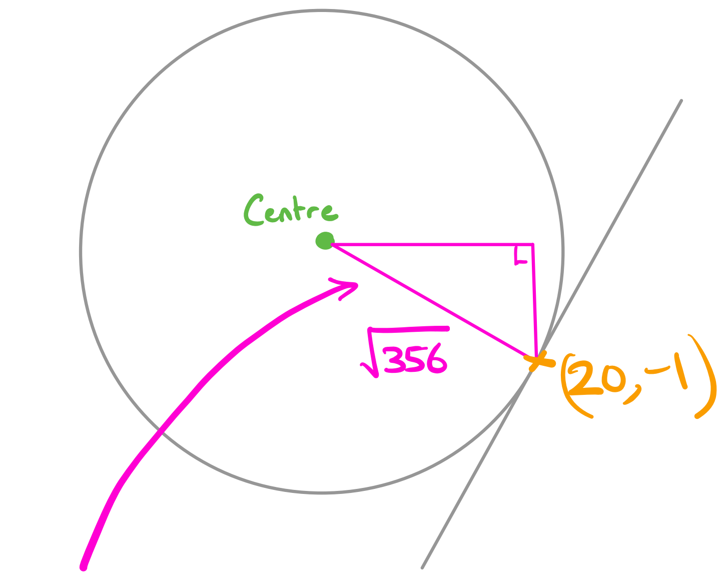
20 A circle has a radius of $\sqrt{356}$. The line with equation $\underbrace{y = \frac{8}{5}x - 33}$ is tangent to this circle at the point $(20, -1)$.

Find the coordinates of the centre of the circle.

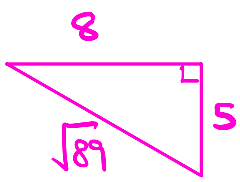
Gradient $= \frac{8}{5}$

[6 marks]

Gradient of radius to $(20,-1)$ is $\frac{-5}{8}$

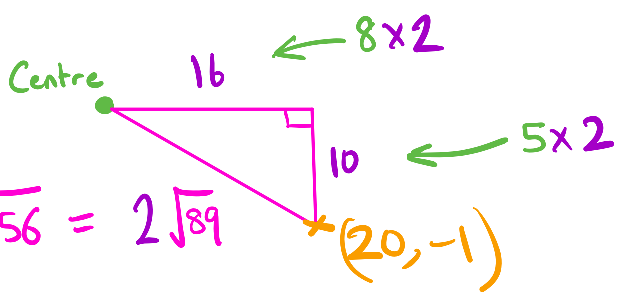


This must be a similar triangle to



for the radius to have gradient $-\frac{5}{8}$

$\sqrt{356} = 2\sqrt{89}$, so we have:



$\sqrt{356} = 2\sqrt{89}$

Answer $(4, 9)$