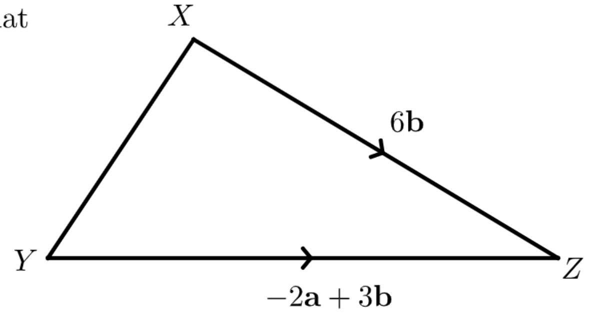
Question 1

Q is the point on XY such that
$\overrightarrow{XQ} = 2\overrightarrow{QY}$

Find the vector $\overrightarrow{ZQ}$
in terms of $\mathbf{a}$ and $\mathbf{b}$.



Question 2

n is a positive integer.
Show that $7n^2 + 29 - (n-1)^2$ is always even.

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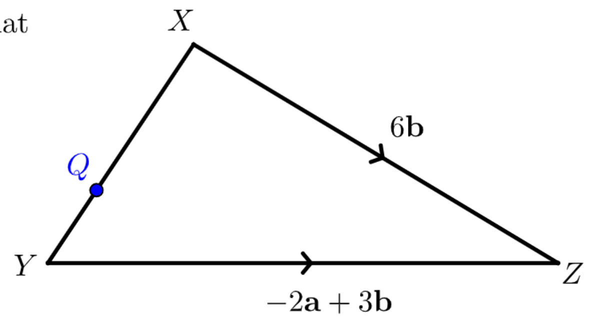

First note that $\overrightarrow{XQ} = \frac{2}{3}\overrightarrow{XY}$.

$$\begin{aligned}\overrightarrow{XY} &= \overrightarrow{XZ} + \overrightarrow{ZY}\\ &= \overrightarrow{XZ} + \left(-\overrightarrow{YZ}\right) = (6\mathbf{b}) + (2\mathbf{a} - 3\mathbf{b}) = 2\mathbf{a} + 3\mathbf{b}\end{aligned}$$

So $\overrightarrow{XQ} = \frac{2}{3}(2\mathbf{a} + 3\mathbf{b}) = \frac{4}{3}\mathbf{a} + 2\mathbf{b}$

Now, $\overrightarrow{ZQ} = \overrightarrow{ZX} + \overrightarrow{XQ}$

$$= -\overrightarrow{XZ} + \overrightarrow{XQ} = (-6\mathbf{b}) + \left(\frac{4}{3}\mathbf{a} + 2\mathbf{b}\right) = \frac{4}{3}\mathbf{a} - 4\mathbf{b}$$

Question 2

n is a positive integer.
Show that $7n^2 + 29 - (n-1)^2$ is always even.

Expanding and simplifying, we get $6n^2 + 2n + 28$
We can write this as $2(3n^2 + n + 14)$
This is always even.